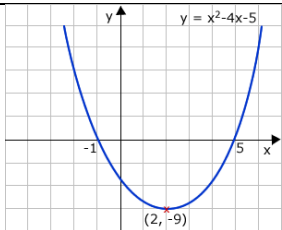
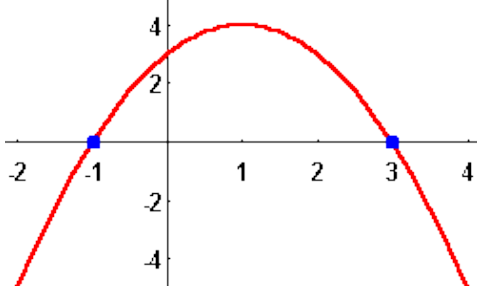



## Topic: Further Quadratics

Topic/Skill	Definition/Tips	Example
1. Quadratic	<p>A quadratic expression is of the form</p> $ax^2 + bx + c$ <p>where <math>a, b</math> and <math>c</math> are numbers, <math>a \neq 0</math></p>	<p>Examples of quadratic expressions:</p> $x^2$ $8x^2 - 3x + 7$ <p>Examples of non-quadratic expressions:</p> $2x^3 - 5x^2$ $9x - 1$
2. Factorising Quadratics	<p>When a quadratic expression is in the form <math>x^2 + bx + c</math> find the two numbers that <b>add to give b</b> and <b>multiply to give c</b>.</p>	$x^2 + 7x + 10 = (x + 5)(x + 2)$ <p>(because 5 and 2 add to give 7 and multiply to give 10)</p> $x^2 + 2x - 8 = (x + 4)(x - 2)$ <p>(because +4 and -2 add to give +2 and multiply to give -8)</p>
3. Difference of Two Squares	<p>An expression of the form <math>a^2 - b^2</math> can be factorised to give <math>(a + b)(a - b)</math></p>	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
4. Solving Quadratics ( $ax^2 = b$ )	<p>Isolate the <math>x^2</math> term and square root both sides.</p> <p>Remember there will be a <b>positive and a negative solution</b>.</p>	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
5. Solving Quadratics ( $ax^2 + bx = 0$ )	<p><b>Factorise</b> and then <b>solve = 0</b>.</p>	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0 \text{ or } x = 3$
6. Solving Quadratics by Factorising ( $a = 1$ )	<p><b>Factorise</b> the quadratic in the usual way.</p> <p><b>Solve = 0</b></p> <p>Make sure the equation = 0 before factorising.</p>	<p>Solve <math>x^2 + 3x - 10 = 0</math></p> <p>Factorise: <math>(x + 5)(x - 2) = 0</math></p> $x = -5 \text{ or } x = 2$
7. Quadratic Graph	<p>A 'U-shaped' curve called a <b>parabola</b>.</p> <p>The equation is of the form <math>y = ax^2 + bx + c</math>, where <math>a, b</math> and <math>c</math> are numbers, <math>a \neq 0</math>.</p> <p>If <math>a &lt; 0</math>, the parabola is <b>upside down</b>.</p>	
8. Roots of a Quadratic	<p>A root is a <b>solution</b>.</p> <p>The roots of a quadratic are the <b>x-intercepts of the quadratic graph</b>.</p>	

9. Turning Point of a Quadratic	<p>A turning point is the <b>point where a quadratic turns.</b></p> <p>On a <b>positive parabola</b>, the turning point is called a <b>minimum</b>.</p> <p>On a <b>negative parabola</b>, the turning point is called a <b>maximum</b>.</p>	
10. Factorising Quadratics when $a \neq 1$	<p>When a quadratic is in the form <math>ax^2 + bx + c</math></p> <ol style="list-style-type: none"> <li>1. Multiply <math>a</math> by <math>c = ac</math></li> <li>2. Find two numbers that add to give <math>b</math> and multiply to give <math>ac</math>.</li> <li>3. Re-write the quadratic, replacing <math>bx</math> with the two numbers you found.</li> <li>4. Factorise in pairs – you should get the same bracket twice</li> <li>5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.</li> </ol>	<p>Factorise <math>6x^2 + 5x - 4</math></p> <ol style="list-style-type: none"> <li>1. <math>6 \times -4 = -24</math></li> <li>2. Two numbers that add to give <math>+5</math> and multiply to give <math>-24</math> are <math>+8</math> and <math>-3</math></li> <li>3. <math>6x^2 + 8x - 3x - 4</math></li> <li>4. Factorise in pairs: <math>2x(3x + 4) - 1(3x + 4)</math></li> <li>5. Answer = <math>(3x + 4)(2x - 1)</math></li> </ol>
11. Solving Quadratics by Factorising ( $a \neq 1$ )	<p><b>Factorise</b> the quadratic in the usual way. <b>Solve = 0</b></p> <p>Make sure the equation = 0 before factorising.</p>	<p>Solve <math>2x^2 + 7x - 4 = 0</math></p> <p>Factorise: <math>(2x - 1)(x + 4) = 0</math></p> $x = \frac{1}{2} \text{ or } x = -4$
12. Completing the Square (when $a = 1$ )	<p>A quadratic in the form <math>x^2 + bx + c</math> can be written in the form <math>(x + p)^2 + q</math></p> <ol style="list-style-type: none"> <li>1. Write a set of brackets with <math>x</math> in and <b>half</b> the value of <math>b</math>.</li> <li>2. Square the bracket.</li> <li>3. Subtract <math>\left(\frac{b}{2}\right)^2</math> and add <math>c</math>.</li> <li>4. Simplify the expression.</li> </ol> <p>You can <b>use the completing the square form</b> to help <b>find the maximum or minimum</b> of quadratic graph.</p>	<p>Complete the square of <math>y = x^2 - 6x + 2</math></p> <p>Answer:</p> $(x - 3)^2 - 3^2 + 2$ $= (x - 3)^2 - 7$ <p>The minimum value of this expression occurs when <math>(x - 3)^2 = 0</math>, which occurs when <math>x = 3</math></p> <p>When <math>x = 3</math>, <math>y = 0 - 7 = -7</math></p> <p>Minimum point = <math>(3, -7)</math></p>
13. Completing the Square (when $a \neq 1$ )	<p>A quadratic in the form <math>ax^2 + bx + c</math> can be written in the form <math>p(x + q)^2 + r</math></p> <p>Use the same method as above, but factorise out <math>a</math> at the start.</p>	<p>Complete the square of <math>4x^2 + 8x - 3</math></p> <p>Answer:</p> $4[x^2 + 2x] - 3$ $= 4[(x + 1)^2 - 1^2] - 3$ $= 4(x + 1)^2 - 4 - 3$ $= 4(x + 1)^2 - 7$
14. Solving Quadratics by Completing the Square	<p><b>Complete the square</b> in the usual way and <b>use inverse operations to solve.</b></p>	<p>Solve <math>x^2 + 8x + 1 = 0</math></p> <p>Answer:</p> $(x + 4)^2 - 4^2 + 1 = 0$ $(x + 4)^2 - 15 = 0$

		$(x + 4)^2 = 15$ $(x + 4) = \pm\sqrt{15}$ $x = -4 \pm \sqrt{15}$
15. Solving Quadratics using the Quadratic Formula	<p>A quadratic in the form <math>ax^2 + bx + c = 0</math> can be solved using the formula:</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>Use the formula if the quadratic does not factorise easily.</p>	<p>Solve <math>3x^2 + x - 5 = 0</math></p> <p>Answer:  <math>a = 3, b = 1, c = -5</math></p> $x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$ $x = \frac{-1 \pm \sqrt{61}}{6}$ <p><math>x = 1.14</math> or <math>-1.47</math> (2 d.p.)</p>